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## **General Equilibrium Theory, its history and its relation (if any) to the Market Economy**

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### **Abstract**

This brief paper poses the question of general equilibrium theories' relevance for analysing the market economy, by investigating the historical evolution of the model and its authors, with the simplifications, and outright re-writing of the framework from Walras, Marshall and Pareto, to Arrow and Debreu. The background shows how the models' focus and contents has changed very subtly over time, leading to big divergences from the original notions of decentralisation and bargaining, to modern centralised price setting and no negotiation. The paper shows how the modern notion of "walrasian equilibrium" is in fact a misnomer, as there is not anything Walrasian about it. Also we note that the Edgeworth box is named wrongly, while the discussion of the relevance of general equilibrium to a market economy takes us down a path of many formulae and few (if any) concrete connections between a market economy and the theory in question.

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## INTRODUCTION.

General equilibrium theory is the attempt of economists to explain the functioning of the market as a whole. It has evolved throughout economic thinking, from the guiding of Adam Smith's invisible hand, and Marshall's multiple market interactions, in the setting of prices. To Walras' Tatonnement endowments, and Pareto's "tastes and obstacles" markets, all the way through Arrow and Debreu's convex preference assumptions, and axiomatic approach to general equilibrium.

In order to illustrate how the analysis can shed light on the workings of a market economy, it seems paramount to understand the path that general equilibrium theory has taken, and how, its underlying assumptions and aims have changed.

In order to facilitate this, the essay will start with a history of general equilibrium theory, and its application. Arriving at the modern Neo-Walrasian approach to general equilibrium, this particular analysis will be explained more thoroughly, focusing on the contributions it has made towards understanding markets, but also investigating its limitations, and problems. Finally a conclusion is drawn, looking at the properties of general equilibrium theory present and past, and the issues it does not address.

Initially though, we must understand the term "market economy", and its application within economics. A free market economy is one wherein all economic agents act independently of intervention<sup>1</sup>, in their own perceived self-interest, with all quantities determined by the laws governing demand and supply, and prices determined by the price-mechanism. A planned economy in contrast, is one where all decisions of quantity and prices<sup>2</sup> are taken centrally, including decisions of distribution. A market economy exists somewhere in between. It has some intervention, but the determination of prices and quantities, rest mainly with the market mechanisms.

Knowing this, we now look at the analysis of a market economy, through the tool we know as general equilibrium theory.

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<sup>1</sup> Intervention is defined as distortions in "market based" prices or quantities due to government influence, producer power or consumer power.

<sup>2</sup> The need for prices in a centrally controlled economy, arises mainly from the need for accounting for output in gdp contexts and for valuation of output.

## THE EVOLUTION OF GENERAL EQUILIBRIUM THEORY.

From the time of Adam Smith's *Wealth of Nations* in 1776, one recurrent theme of economic analysis has been the remarkable degree of coherence among the vast number of individual and seemingly separate decisions about the buying and selling of commodities. In everyday, normal experience, there is something of a balance between the amounts of goods and services that some individual want to supply and the amounts that other, different individuals want to sell. Would-be buyers ordinarily count correctly on being able to carry out their intentions. (Arrow 1974, p. 253)

Marshall (1890) and Walras (1889) each wrote a defining work on economics, and their understanding of how the economy would arrive at a “normal equilibrium”, where all markets cleared. Both authors agree that market forces do exist, and that they work, almost perfectly at most times, even though equilibrium is not constant.

The position of normal equilibrium at any time is rather to be regarded as one towards which the forces of demand and supply at the time are tending, than as one that is ever actually attained  
(Marshall, 1890, Book VII, p. 540)

The market is like a lake agitated by the wind, where the water is incessantly seeking its level without ever reaching it.  
(Walras 1889, in Jaffé ed. 1954, p. 380)

As pointed out by Clower (1996), their approach is different. Walras looks at Net excess demand, but Marshall analyses the phenomenon in terms of prices of demand and supply. None-the-less, the conclusions reached by both authors are similar, if we separate the achievement of exchange equilibrium, through different buyers outbidding each other, and that of production (supply) equilibrium, which is found through variations in quantity.

In both authors, exchange is conducted by traders, and transaction prices operate as signals which serve as the ‘active force’ for producers to change quantities to supply. Thus, a first conclusion is that equilibrium in exchange is arrived at by means of price adjustments, and equilibrium in production by quantity adjustments in both

authors. And second, that in both Marshall and Walras, the determination of prices is carried out by traders. (Costa, 1998, p. 35)

They both find that the underlying currents in the economy, can lead to efficient setting of prices and determining of output, all by themselves. Further, even though Walras is vague on the issue of establishing price<sup>3</sup>, they find that price determination is undertaken in the market economy<sup>4</sup>.

In 1906, Vilfredo Pareto wrote his *Manual of Political Economy*, and it was his analysis of the general equilibrium model which would dominate the study for the next fifty years to come.

The principal subject of our study is economic equilibrium. We shall see shortly that this equilibrium results from the opposition between men's tastes and the obstacles to satisfying them. Our study includes, then, three distinct parts: 1° the study of tastes; 2° the study of obstacles; 3° the study of the way in which these two elements combine to reach equilibrium.

(Pareto, 1906, p. 106)

It is with this “tastes-and-obstacles” approach that the Paretian School attempts the problem of general equilibrium, and it focuses on agent optimization for price takers (not price setters, or negotiators as earlier) in a multi market environment. The tools of differentiability and efficiency are corner stones of the Paretian school, and we find that the question of suppliers meeting demand of traders (before price setting) is assumed to be irrelevant, as agents perform the dual role of producing and trading.

While the Paretian School dominated the discipline, the German speaking world, under the name of the Vienna Colloquium was heavily influenced by Cassel's 1918 revival of Walras' original theories of equilibrium. The Walras-Cassel model found,

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<sup>3</sup> Walras concept of Tatonnement, means that no goods/endowments are storable, and so agents have a fixed endowment each time, and the issue of non-trading in disequilibrium, means that trading outside equilibrium is unfeasible, but does not infer that no negotiation takes place, to find a price.

<sup>4</sup> For Walras, exchange equilibrium would occur when excess demand equalled zero, whereas Marshall's system stated that when the price of supply equalled that of demand, markets would clear. These two views, are complementary, rather than opposites (Costa 1998).

General equilibrium is defined as a set of factor prices and output prices such that the relevant quantities demanded and supplied in each market are equal to each other, i.e. both output and factor markets clear. (Fonseca & Ussher; 2004b, p.1)

The school introduced the concepts of complimentary slackness, which lead to the proof of a unique equilibrium for the static analysis of Walras equilibrium, using the axiom of weakly revealed preference. We should note the shift away from dynamic equilibriums in the original Walras papers.

During the 1930's, there was increased interest in the Paretian School, and a mild Marginalist revival took place (ibid, 2004f), through the work of Hicks (1939) amongst others, who published through the Cowles Commission.

[The head of the Economics department] said to me, “you read Italian? You ought to read Pareto”. So it was reading the *Manuale* which started me off on economic theory. I was deep in Pareto before I got much out of Marshall.

(Hicks, 1979, *The formation of an Economist*)

As the Second World War broke out, the Vienna Colloquium was disbanded, but some of its members re-emerged in the United States, primarily at Chicago University in particular, where the Cowles Commission was founded in 1939<sup>5</sup>, with the maxim, “Science is Measurement”. The Cowles commission aimed to link mathematics and economics (Fonseca & Ussher 2004g), and benefited immensely from the influx of European economists who had worked with Walrasian equilibrium theory.

The combination of the Cowles Commission and the Paretian School, led to what we today consider as the basics of general equilibrium theory, the Neo-Walrasian School.

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<sup>5</sup> The Cowles Commission later relocated to Yale in 1955, and was re-named as the Cowles Foundation. Some prominent economists associated with the foundation are: Arrow, Debreu, Stiglitz, and Tobin.

THE NEO WALRASIAN GENERAL EQUILIBRIUM – (CONTEMPORARY MODEL BASICS)

It is worth pointing out that in this particular study our authors have abandoned demand and supply functions as a tool for analysis, even as applied to individuals... [The problem] has been reformulated as one of proving that a number of maximizations of individual goals under interdependent restraints can be simultaneously carried out.

(Koopmans, 1957, p.60)

It is this school of thought, which underpins most of modern economic analysis and modelling of a general equilibrium<sup>6</sup>. By combining the “tastes-and-obstacles”, and individual economic optimising approach of Pareto, but using the weak axiom of preference from the Vienna Colloquium (rather than that of Pareto’s differentiability), and relying on convex preferences, has allowed the Neo-Walrasians school, to redefine the study of equilibrium.

The assumptions underpinning the simple stable equilibrium, are those of a perfectly competitive market, where firms set no output outside equilibrium, and the use of a Walrasian auctioneer<sup>7</sup> is needed to solve the co-ordination problem, which exists as there is no place for price negotiation, i.e. exchange equilibrium is not achieved through the market, rather it is accomplished by the auctioneer.

At its simplest, this is a model of consumers and producers. Income is assumed not to exist, as this is an exchange model, and is therefore implied within given endowments. Goods are distributed by time, place and ‘state of nature’, but only as forward looking observations, as all markets for future and present exchange and supply must clear at the beginning of the world, as trade is assumed not to take place outside equilibrium<sup>8</sup>.

Using relative prices to establish equilibrium, the concern of the model is to achieve efficient distribution of goods in the economy (maximizing utility), and as a result,

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<sup>66</sup> The concern of the literature shifted from the first half of the century to the second, from a focus on social optimum outcomes and market efficiency with the Paretians, to the Neo-Walrasian question of whether or not equilibrium existed, and if so, is equilibrium stable.

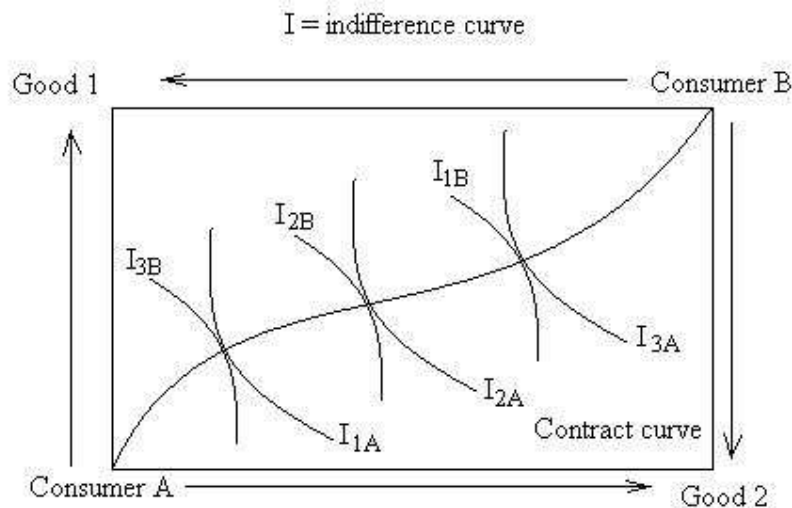
<sup>7</sup> This concept is not taken from Walras theory, rather it is an inference, that there be an entity, which collects all demand and supply information, and then attempts to set a price which will clear the markets.

<sup>8</sup> This is an after-effect of the Walrasian Tatonnement argument, but is addressed later.

excess demand curves and aggregate demand must be continuous, to solve a Brouwer fixed point equilibrium. This issue and the problem of imperfect competition<sup>9</sup>, is solved by assuming perfect competition and a Cobb Douglas demand function.

Consumer preferences are assumed to be continuous and monotonic, to assure a weak Pareto efficient outcome<sup>10</sup>, and all agents are price takers. Under the assumption that all goods are desirable<sup>11</sup>, we can then demonstrate that a Walrasian<sup>12</sup> equilibrium exists if the allocation of goods is feasible, and each agent acts as a utility optimizer.

We should note that all consumers are homogenous, in that they all have convex utility curves, and that the problem of aggregating utility across the economy is not addressed. Following the first and second theories of welfare, which are proved under these assumptions (Varian 1996), we can show that all competitive equilibria are Pareto efficient, and that an initial redistribution can lead to a Pareto efficient outcome. This is usually illustrated by using an Edgeworth Box<sup>13</sup>:



<sup>9</sup> Demand curves faced by collusive oligopolies for example are kinked.

<sup>10</sup> Varian (1992) p. 323: "A feasible allocation  $x$  is a weakly Pareto efficient allocation if there is no feasible allocation  $y$  such that all agents strictly prefer  $y$  to  $x$ ."

<sup>11</sup> If not, there may be free goods in the economy, ruining the possibility of a general equilibrium.

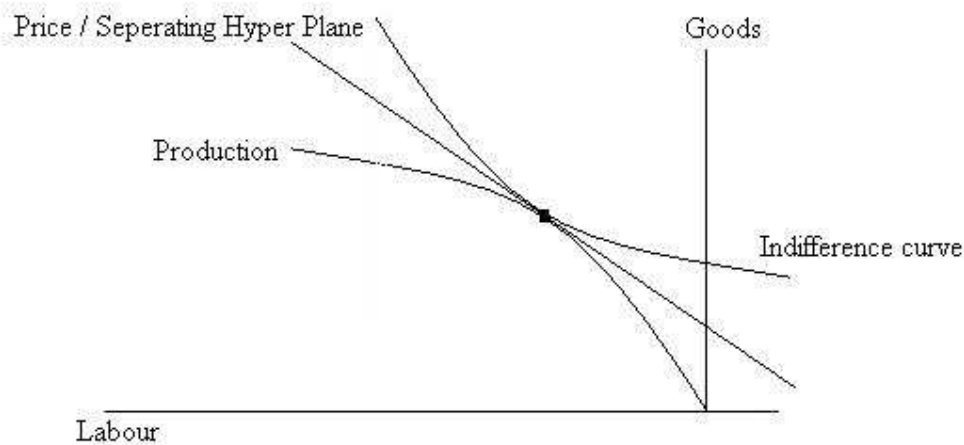
<sup>12</sup> This is defined as a static equilibrium when "the allocation is feasible, and each agent is making an optimal choice from his budget set" (Varian, 1996, p. 325) – not to be confused with Walras' original notions of (dynamic) equilibrium.

<sup>13</sup> Ironically, the Edgeworth box was not invented by Edgeworth, but was first drawn in Pareto's *Manuale* (1906), the wrongful naming was dubbed "intellectual gangsterism" by Maurice Allais in 1946 (Fonseca & Ussher, 2004d), but we shall use its typical name, not Allais' justified "Pareto box".

An Edgeworth box with two consumers and two goods, show how all contract curve intercepts could be possible equilibria. However, the underlying assumptions are:

1. Exchange Efficiency, goods go to those who value them the most ( $MRS_1=MRS_2$ )<sup>14</sup>;
2. Production Efficiency, Marginal rate of technical substitution equal for all firms ( $MRTS_1 = MRTS_2$ );
3. Product Mix Efficiency, Goods produced correspond to those wanted, i.e. Marginal Rate of Transformation = MRS (Stiglitz, 2000).

The theory can be extended to producing firms, which have to be perfectly competitive, and thereby, price takers. Assuming well behaved<sup>15</sup> Production functions, that can be aggregated, equilibrium can be shown for the simple stochastic case, known as the “Robinson Crusoe example”, but more importantly the system can be generalised for multiple firms and consumers, using a separating hyper plane, to set a price vector which will clear all markets<sup>16</sup>.



Under the assumptions of Diminishing marginal returns<sup>17</sup>, convex indifference curves and isoquants, continuity, and the possibility of aggregating all utilities and production, a price can be set to clear all markets simultaneously.

<sup>14</sup> MRS = Marginal Rate of Substitution.

<sup>15</sup> Strictly Convex most critically. For full list of equilibrium conditions see Varian (1996, p. 345) and Debreu (1959) for proofs.

<sup>16</sup> A note needs adding, that when the Neo-Walrasian approach refers to all markets, it is assumed that everything in the world can be reduced to a market form, with a specific outcome resting on prices and supply & demand. I.e. markets for externalities and uncertainty are assumed to be endogenized.

<sup>17</sup> Of both productivity and utility.

## PROBLEMS OF THE GENERAL EQUILIBRIUM MODEL

Multiple criticisms has been levelled at the model, initially for its lack of dynamics,

Optimizing responses of economic agents are simultaneously feasible only if the proper prices are already known to them. But these prices must somehow themselves be the result of the same responses. (Koopmans, 1974, p.327)

Its treatment of time, requires traders and suppliers to know all possible outcomes, for all times, and successfully develop markets for possible risks and uncertainties<sup>18</sup>. Utility is additive and discounted over time, (Varian 1996), two things which may not be suitable nor realistic. Further; if the time span is expanded to infinity, the first theory of welfare fails unless the Neumann-Morgenstern Axioms<sup>19</sup> hold.

The fact that perfect competition does not exist, and the effect that other industry structures have, has been addressed through the theory of the core, stating that, as the number of agents grow, groups form coalitions. A feasible allocation will exist within the core<sup>20</sup>, and assuming one allocation cannot be improved by any coalition formed, then competitive equilibrium is the outcome. This depends on assuming that there are no costs to forming a coalition, and that information about coalitions is perfect (ibid), also the convexity assumption is crucial.

Issues of production externalities have not been resolved within the model. Arguments of introducing Pigovian taxes<sup>21</sup>, or internalizing the externality cost, fails to recognize the problems of consumption -and public good externalities (ibid). And the issue of imperfect information is unsatisfactorily dealt with, by assuming zero profits (ibid). The critiques are many, but the question is whether general equilibrium theory is still useful for analysing the market economy.

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<sup>18</sup> Uncertainty markets seem hard to control for, as outcomes are unknown, and it is unknown if the outcomes can or will happen.

<sup>19</sup> From Ussher & Fonseca (2004h) "In the von Neumann-Morgenstern hypothesis, probabilities are assumed to be "*objective*" or exogenously given by "Nature" and thus cannot be influenced by the agent. However, the problem of an agent under uncertainty is to *choose* among lotteries, and thus find the "best" lottery."

<sup>20</sup> 'the core' is the area of the contract curve, which separates the two indifference curves (in case of two good, two consumer world), and it is this which causes equilibrium not to be reached.

<sup>21</sup> Where a price is assigned to each externality, and compensation is paid.

## GENERAL EQUILIBRIUM AND THE MARKET ECONOMY

General equilibrium theory was and is, for all practical purposes intended to show if and how the economy as a whole converges towards equilibrium, and then to discover the mechanism by which it does so. Modern general equilibrium theory fails in all practical connections to this, as it from the onset does not consider money as a part of an economy, and constrains itself to perfectly competitive markets only.

*One.* Does the model of competitive equilibrium in its simplest form represent one useful pure and special case, one valuable foothold for a steep climb? My answer: Yes.

*Two.* Can we as yet evaluate the merit or promise of the various ramifications of the theory in recent literature? My answer: I find it hard to assess this fascinating blend of high achievements, challenging starts and possible dead ends.

(Koopmans, 1974, p.325)

The theory of *Competitive Equilibrium* does indeed provide a useful benchmark from which we can show that market forces work to maximize utility and output in an economy. However there is a complete lack of price determination and money within the model, which means the modern models, cannot tell us anything of the efficient setting of prices. In fact, as opposed to Walras' and Marshall's decentralized price negotiations, which gave way to an equilibrium, both for output and price, we now have a centralized price setter (the Walrasian Auctioneer), which attempts to balance input and output, with seemingly no intervention from the traditional market forces.

Here the function of finding equilibrium prices is clearly viewed as requiring a central price-maker, the 'auctioneer'. Strangely, although Walras had started from decentralized bargaining, and general equilibrium analysis is barren in the understanding of adjustment processes. (Costa, 1998, p.35)

*To the question: "Who chooses prices?":* In the formal theory at least no-one. They are determined on social institutions known as markets, which equate supply and demand.

(Arrow, 1994, p.4)

It is part of the auctioneer's job to freely disseminate offers to buy and sell. (Arrow & Hahn, 1971, p.329)

In fact, we find that even though it was Arrow's (1959) ambition to actually allow prices to be determined within the model, through exchange, Costa's finds that,

Arrow does not fill the 'logical gap' in the usual formulations of the theory of the perfectly competitive economy that there is no place for a price decision as there is with respect to quantities. (Costa, 1998, p.114)

We have shown how the classical models of general equilibrium, fostered the notion of converging prices and efficient allocation of output, due to market forces. Whereas modern, Neo-Walrasian equilibrium analysis seems only useful as a benchmark case for the purely theoretical world, and its application to the understanding of the market economy as a whole, has been diminished by the numerous (unrealistic) assumptions laid on the *specific case*, which 'only' gives Pareto efficient outcomes. Further, the study of equilibrium in modern times, has failed to shed any new light on the issue of the market economy, except for its failure to coincide with perfect competition.

[The Neo-Walrasian approach to the] analysis of competitive markets in a general equilibrium conceptualization was a dominant body of formal models whose unrelenting aim at 'perfection' was achieved at the cost of missing the empirical mark. The attention to equilibrium states with its concentration on proofs of existence, virtual stability and uniqueness, led the analysis of competitive markets, bargaining feasibility of trading and, in general, of processes of adjustment to the sidelines of theoretical inquiry. (Costa, 1998, p.52)

In attempting to answer the question 'Could it be true?', we learn a good deal about why it might not be true.  
(Arrow & Hahn, 1971, p. vii)

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